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A NONLINEAR MODEL FOR OPTIMIZATION OF SMALL RESERVOIR IRRIGATION SYSTEMS FOR SEMI-ARID TROPICS[1/]

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1. INTRODUCTION

Water harvesting into small reservoirs is an age old concept for storing water for Various uses in the semi-arid tropics of the world. Their use is presently being expended in many developing countries including Brazil, due to a variety of socio-economic and technical reasons. However, most of the present day guidelines for planning, designing, location and storage capacity of these small reservoirs have been based on traditional design principles with out any optimization criteria. The first effort on optimization of the small reservoir systems was carried out recently by

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Sharma (1981). The present paper is based on this work of Sharma (1981) which is now in press (Sharma and Helweg; Helweg and Sharma, in press). The purpose of the present paper is to make this work available immediately to the brazilian collegues for their use. This work will also prove handy as a base for their future research projects which have been outlined separately by Sharma (1982). In the present paper, first the concept of an elevated inlet type small reservoir system is presented which is followed by the development of the basic model and its solution technique. Finally a typical example of optimization of small reservoir system is given for Alfisols in Hyderabad region of semi-arid India which is based on the data collected at the International Crops Research Institute for Semi-Arid Tropics (India). At the end, suggestions for its applicability to various climatic zones (Sharma, 1982) of North-East Brazil is discussed.

2. SMALL RESERVOIR IRRIGATION SYSTEM OPTIMIZATION: MODEL DEVELOPMENT

2.1. THE CONCEPT

The majority of traditional small reservoirs are shallow, occupy large tracts of land, serve lesser land and lose high proportion of their storage capacity due to evaporation and seepage. The small reservoirs recommended nowadays, also known as farm ponds, are usually dug type. They have a very poor storage to excavation ratio* (approximately equal to 1), hence make the water expensive. Cluff (1977) has suggested compartmenting of shallow reservoirs and then concentrating the water by pumping in to the deeper compartment, for reducing evaporation losses. Sharma (1981) has suggested the principle of elevated inlet type of reservoirs for more efficient storage of water into small reservoirs which also increases the storage to excavation ratio of reservoir thus reducing the cost of construction considerably.

*Storage to excavation ratio is defined as the ratio of capacity of a reservoir and the excavation required to build it.

To construct an elevated inlet type of reservoir, runoff in a catchment can be intercepted at a higher elevation. The runoff is then led by an elevated earthen channel to the reservoir below so that water is partly stored above ground and partially in the excavated portion below ground level. The only excavation required is one needed to build the dyke equal in height to the elevation of the point where runoff is intercepted. This concept is demonstrated in Fig. 1 and has been successfully tried by Sharma and Kampen (1976) at the International Crops Research Institute for Semi-Arid Tropics (ICRISAT), Hyderabad, India. Storage to excavation ratio of 2.5 and as high as 2.9 were obtained in a 15 ha. Catchment varying in slope from 1.25% to 3% (Sharma and Kampen, 1977). This system developed at ICRISAT (Sharma and Kampen, 1977) is not an optimum and suffers from location specificity. However, the concept of elevated inlet type reservoir can be universally used in areas having sloping topography. The following sections give the analytic background for optimization of such systems though the methodology developed here for optimization is valid for any other shape also.

While there is no doubt that there will be savings in evaporation losses due to reduction in exposed water surface area in the case of elevated inlet type reservoirs, the seepage losses are a function of depth of water also. But if a swelling clay liner is used this effect can be offsetted. Seepage from an earth-lined reservoir over an alluvial soil in an arid environment fits Bouwer's (1969) descripition of a channel with a clogged soil at its perimeter. In this case the underlying soil is unsaturated and the flow according to Sposito (1975) is controlled by the negative soil-water pressure in the underlying drier material. Sposito finds that the depth of water in a shallow reservoir may not have much

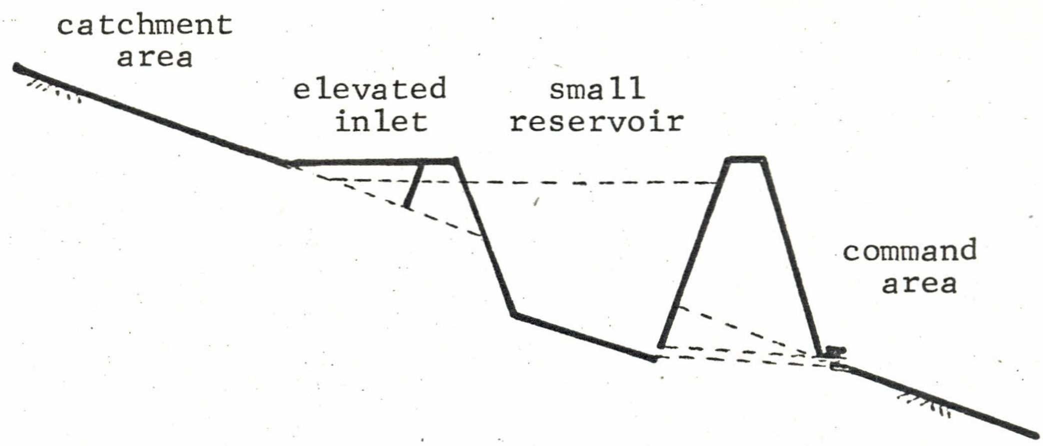

Fig. 1 : Conceptualization of an elevated inlet type small reservoir

effect on the seepage particularly if a swelling-clay liner is used. The effect of the swelling offsets the gravity forces, leaving the soil pressure in the underlying unsaturated material as the controlling factor in seepage. This finding indicates that in applying the concept of the elevated inlet type reservoir system with an earth-liner there should be a reduction in seepage loss due to concentrating the water and reducing the area contributing to seepage.According to Sposito's findings this reduction should be greater than the increase in seepage caused by increasing the depth by an elevated inlet.

2.2..MODEL FORMULATION

There are three separable optimization problems in designing a small reservoir or tank irrigation system. The first has to do with optimal tank design, the second has to do with the optimal storage capacity and the third with the location of the tank in the catchment. For theoretical analysis here, some basic assumptions are being made.

2.2.1. ASSUMPTIONS

1. Rainfall - runoff relationship for a catchment is known.
2. The general topography of the catchment is sloping but for simplification the land under a tank is considered flat. Other topographic features at the location of a tank e.g. depressions, man made holes, gullies etc. can be considered in the solution but are being ignored to avoid complexities in analysis.
3. The most economic shape of a tank is an inverted truncated cone with circular cross section and trapezoidal dyke. This will permit least length of the dyke (thus least excavation required), take least area

under it, will have least exposed surface available for evaporation and minimum wetted perimeter (thus less seepage), compared to other shapes. However, the general theory and approach holds good for any definable shape.

Additional assumptions as and when made are explained at the point of interested in the text.

2.2.2. SMALL RESERVOIR GEOMETRY:

To illustrate the approach to this problem, consider a simplified design of a tank constructed on level ground as in Fig. 2,1 a hypothetic cross sectional sketch of a truncated cone shaped tank. Let:

V_1 = Volume of excavation

V_2 = Volume of dyke

V_3 = Volume of tank above ground level

V_t = Total volume of tank

h_1 = Design depth of excavation

h_2 = Design height of water above ground level

1 : n = Side slopes of dyke

r_0 = Inside radius of tank at the bottom

r_1 = Inside radius of tank at ground level

r_2 = Inside radius of tank at design height

GL = Ground level

Further assume the top width of the dyke is known, say 2 m, and free board to be provided as 1 m.

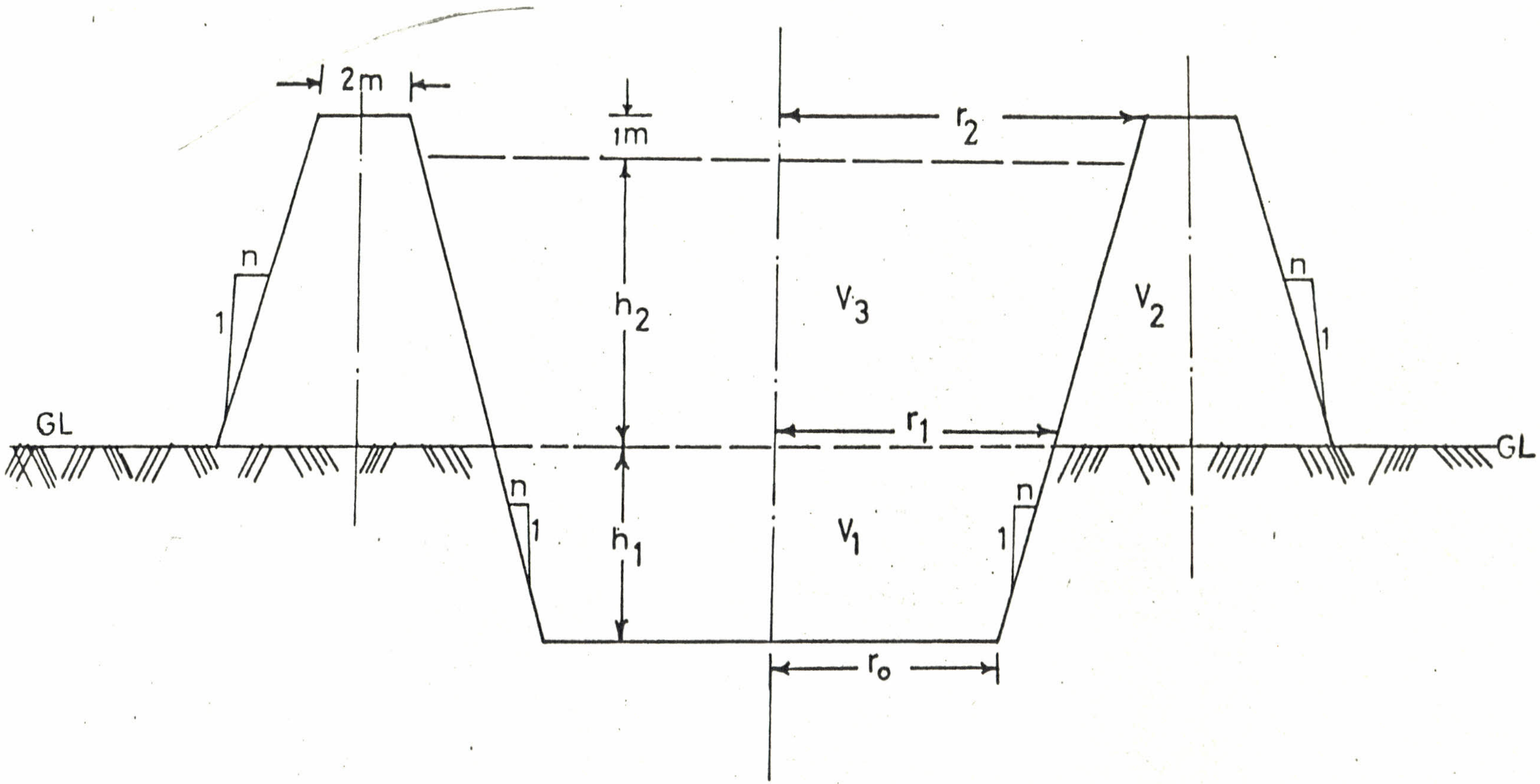

Fig. 2 :Hypothetic cross sectional sketch of a truncated cone shaped tank.

The formulae for V_1 and V_3 are straight forward solid geometry and V_2 can be found by Theorem II of Pappus. Using the notations in Fig. 2, the results are:

$$V_1 = \frac{\pi}{3} h_1 (3r_1^2 - 3nr_1h_1 + n^2h_1^2) \qquad (1)$$

$$V_2 = 2\pi (r_1 + nh_2 + n + 1)(nh_2 + n + 2)(h_2 + 1)$$

$$= 2\pi (n^2h_2^3 + nr_1h_2^2 + 3n^2h_2^2 + 3nh_2^2 + 2nr_1h_2 + 2r_1h_2 + 3n^2h_2 + 6nh_2 + 2h_2 + nr_1 + 2r_1 + n^2 + 3n + 2) \qquad (2)$$

$$V_3 = \frac{\pi}{3} h_2 (3r_1^2 + 3nr_1h_2 + n^2h_2^2) \qquad (3)$$

2.2.3. OPTIMUM SMALL RESERVOIR STORAGE CAPACITY, LOCATION AND DIMENSIONS

The optimum storage capacity and location of a small reservoir for an irrigation system as a whole and hence its dimensions, on a small watershed basis are decided on the following two bases.

2.2.3. (A) UNCONSTRAINED SMALL RESERVOIR IRRIGATION SYSTEM OPTIMIZATION

Consider that there is an unlimited catchment area available for producing unlimited runoff to be stored in a tank and that there is an unlimited command area available for irrigation of any given crop or cropping system. This situation is being termed 'unconstrained' since there are no physical limits being imposed on the optimum storage capacity or location of a tank. In such a case, for regions where water is a precious commodity e.g. semi-arid tropics, the goal of an optimum irrigation system should be to achieve the highest possible water utilization efficiency - in turn maximum net benefits from a system of irrigation. Further more, it has clearly been demonstrated by Sharma (1981) that minimization of excavation only or minimization of total cost of excavation only will not lead to an optimal solution. The objective function of a small reservoir optimization model, as explained earlier, should maximize the net benefits derived from the harvested water after accounting for seepage and evaporation losses. The net benefits from small reservoir irrigation systems are calculated as:

Net benefits = Total benefits - Total costs

Total benefits = (Benefits from crop production due to irrigation/unit of water)

X

(Volume of the reservoir - Seepage losses - Evaporation losses)

Total costs = Total cost of construction of a reservoir

+

Total cost of land occupied by the reservoir

Therefore, the objective function is:

$$\max_{\underline{V}} \; \{B_p \, (V_t' - S_p - E_v) - (C_u V_1 + 0.05 C_u V_1 + C_L A_R)\} \qquad (4)$$

where,

B_p = present worth of benefits per unit of water, Rs./m^3

V'_t = optimum storage capacity of the reservoir, m^3

S_p = cumulative seepage loss, m^3

E_v = cumulative evaporation loss, m^3

C_u = unit cost of excavation, Rs./m^3

C_L = cost of land occupied by the reservoir, Rs./ha.

A_R = area occupied by the reservoir, ha.

The methodology for determination of various components of benefits and costs is discussed here.

(i) DETERMINATION OF BENEFITS

The benefits of a system of irrigation depend on the yield of various crops being irrigated by the system. The yield of a crop is a function of the quantity of water available to it, if all other factors of production e.g. seeds, fertilizers, cultural practices and management are kept constant. The accurate data on water production functions is hard to find in many developing countries although research work has been reported on the topic of yield response to water in the past 25 years or so (for references see Sharma, 1981). Often yield response to amount of irrigation water applied relationships are available instead of yield response to total water use functions. The latter only can accurately predict the true effect of small quantities of supplemental irrigation (or deficit) on yield of a crop. For the purpose of this model, as an example, some recent data from ICRISAT

(Annual Report, 1977-78) for postmonsoon season CSH-1 sorghum on Alfisols was utilized. Though any crops or cropping system may be closen, sorghum was chosen due to its pronounced response to irrigation. In scarity of many observations, it was decided to use these to develop water production function of quadratic nature of the following form which needs only 4 or more observations to be statistically valid:

$$Y = A_1Q^2 + A_2Q + A_3 \qquad (5)$$

where,

Y = yield of a crop, Kg/ha.

Q = water use, m

A_1, A_2 and A_3 = regression coefficients.

For postmonsoon CSH-1 sorghum on Alfisols this equation will be (Sharma, 1981):

$$Y = -399.025Q^2 + 227.80Q - 16.60 \qquad (6)$$

Given this, the benefits of irrigation;

$$B = P \times Y - \text{Cost of Inputs and Operations}$$

or, $$B = P(A_1Q^2 + A_2Q + A_3) - (CI + L \times N_0)$$

where,

Y = yield of a crop, kg/ha.

B = benefits, Rs*/ha.

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* One US $ ≃ 9 Indian Rupees, as per January 1982 exchange rate.

P = existing market price of crop, Rs./100 kg.

CI = fixed cost of inputs e.g. seeds, fertilizers, tillage operations and cultural operations, Rs./ha.

N_0 = number of irrigations.

L = cost of labor for each irrigation, Rs/ha.

The total benefits can now be determined by multipliyng the price of crop and yield obtained at various levels of water use (equation 6). The maximum yield is obtained at 28 cm of water use from equation 6.

Now assuming the life of irrigation project is N_y years and the benefits of irrigation, B, are a uniform annual series spread over N_y years, the present worth of the benefits,

$$P_w = B(P_w/A,\ i\%,\ N_y) = B\,\frac{(1+i)^{N_y} - 1}{i(1+i)^{N_y}}$$

where,

$(P_w/A,\ i\%,\ N_y)$ is the compound interest annual uniform series discount factor and i is the annual discount rate. This discount rate may be different from standard bank interest rates as agricultural development interest rates, in countries like India and Brazil, are less in order to encourage rural development, particularly so in dry and semi-arid regions.

The present worth of irrigation benefits per unit of water use for a given cropping system in an irrigation project can therefore be summarized as:

$$B_p = (P_w/A,\ i\%,\ N_y)\,\frac{\eta}{Q}\,\{P(A_1Q^2 + A_2Q + A_3) - (CI + L \times N_0)\} \qquad (7)$$

where,

η = project irrigation efficiency (assumed 80%)

Incedently, the term $(1/Q)(A_1Q^2 + A_2Q + A_3)$ is the water utilization efficiency.

For determining the total benefits, effective storage capacity of the reservoir available for irrigation is required. The effective storage capacity V' of a reservoir;

$$V' = V_t' - S_p - E_v$$

where V_t' is the total storage capacity for unconstrained case and is to be optimized by the model. The cumulative seepage, S_p and cumulative evaporation, E_v are directly proportional to the wetted perimeter and exposed top surface area of a reservoir respectively, presuming that the effect of depth of water on seepage loss is offsetted by appropriate seepage liners. From Figure 2 (assuming dyke slope as 1:1), thus cumulative seepage;

$$S_p = SN_p\pi\{\frac{1}{\sqrt{2}}(2r_1 - h_1 + h_2)(h_1 + h_2) + (r_1 - h_1)^2\} \qquad (8)$$

where,

S_p = cumulative seepage, m^3

S = average daily seepage rates, m/day

N_p = the period during which water stays in a reservoir after being filled and before being empited, days and cumulative evaporation;

$$E_v = \frac{\pi}{2} EN_p \{(r_1 - h_1)^2 + (r_1 + h_2)^2\} \tag{9}$$

where,

E = Average daily evaporation rate during the period N_p, m/day

E_v = Cumulative evaporation losses, m^3

Finally,

$$\text{Total Benefits} = B_p(V_t' - S_p - E_v) \tag{10}$$

(ii) DETERMINATION OF COSTS

The total cost of a small reservoir irrigation system consists of its construction cost (C_uV_1), operations and maintenance cost ($0.05C_uV_1$) and the cost of the land occupied by the reservoir (C_LA_R). In equation 4;

$$\text{Total cost} = C_uV_1 + 0.05C_uV_1 + C_LA_R \tag{11}$$

the cost of operations and maintenance of the reservoir ($0.05C_uV_1$) has been assumed to be 5% of the initial cost of construction on a flat basis. The unit cost of land C_L, can be determined by a quick survey of the farmers in the project area. The land to be occupied by a small reservoir is a function of both r_1 and height of reservoir h_2. This can be expressed from Fig. 2 for a reservoir of 1:1 dyke slopes, 2 m top width and 1 m free board as:

$$A_R = \pi(r_1 + 2h_2 + 4)^2 \tag{12}$$

Cost of excavation C_u is a nonlinear function of lead and lift. As depth (lift) increases, the cost of digging also increase. Similarly as distance (lead) of hauling the earth increases, the cost of building a dyke also increases. The standard schedules of rates for all type of construction activities including excavation are available with appropriate governmental agencies. Intuitively the additional cost of excavation due to lead and lift is of quadratic nature. The generalized cost function can be written as:

$$C = FI + C_r + C_h$$

where,

FI = Fixed minimum cost

C_r = Incremental cost over FI due to increase in lead

C_h = Incremental cost cover FI due to increase in lift

In case the reservoir construction;

Average lead = r_1

Average lift, h = $(h_1 + h_2)/2$

As an example, the standard rates for excavation for 10 m^3 of earth at various leads and lifts for Hyderabad, Ranga Reddy, Nalgonda and Mahboobnager districts in Andhra Pradesh in Semi-Arid India, as used by their irrigation and power department yields the following relationships:

$$C_r = -0.00037r_1^2 + 0.0686r_1 - 0.22833 \quad (13)$$

$$C_h = -0.03409h^2 + 0.73773h - 0.91363 \quad (14)$$

and;

$$FI = 24.2$$

thus unit cost of construction C_u;

$$C_u = (FI + C_r + C_h)/10 \quad (15)$$

By now all the cost and benefits of a small reservoir irrigation system have been quantified, so the 'unconstrained' small reservoir irrigation system optimization model can now be formulated as:

$$\underset{r_1 h_1 h_2}{MIN} \quad -\{B_p(V_t' - S_p - E_v) - (1.05C_uV_1 + C_LA_R)\} \quad (16)$$

Subject to:

1. Excavation and dyke construction constraint;

$$V_1 = V_2 \quad (17)$$

2. Storage capacity constraint;

$$V_1 + V_3 = V_t' \quad (18)$$

3. Non negativity constraint;

$$r_1, h_1, h_2 \geq 0 \quad (19)$$

where V_t' is the unknown optimum size of reservoir and can be expressed from Fig. 2 for a dyke of 1:1 slopes as:

$$V_t' = \frac{\pi}{3}(h_1 + h_2)\{(r_1 - h_1)^2 + (r_1 + h_2)^2$$

$$+ (r_1 - h_1)(r_1 + h_2)\} \qquad (20)$$

It is to be noted that the terms S_p, E_v, C_u and A_R are all functions of r_1, h_1 and h_2 as expressed in equations (8), (9), (15) and (12) hence will effect the optimum size V_t'. Thus the optimum design dimensions r_1, h_1 and h_2 obtained from above model would be the values adjusted for minimizing seepage, evaporation, cost of excavation, cost of land occupied by the reservoir and will be for a storage capacity of the small reservoir that will maximize net benefits from supplemental irrigation.

In case h_1 and h_2 become restrictive due to soil depth (h_1) restrictions or availability of gravity drop (h_2) for intercepting runoff, these restrictions can easily be imposed as additional constraints on the model as:

$$h_1 \leq SD \qquad (21)$$

$$h_2 \leq DG \qquad (22)$$

where

SD = depth of soil, m

DG = gravity drop available (h_2) within a reasonable distance near the location of a reservoir, m.

The optimum location of the reservoir for this 'unconstrained' case can now be easily determined, knowing V_t'. The reservoir will be located just under the catchment area which will produce V_t' quantity of runoff.

$$A_c' = \frac{V_t'}{R_u} \tag{23}$$

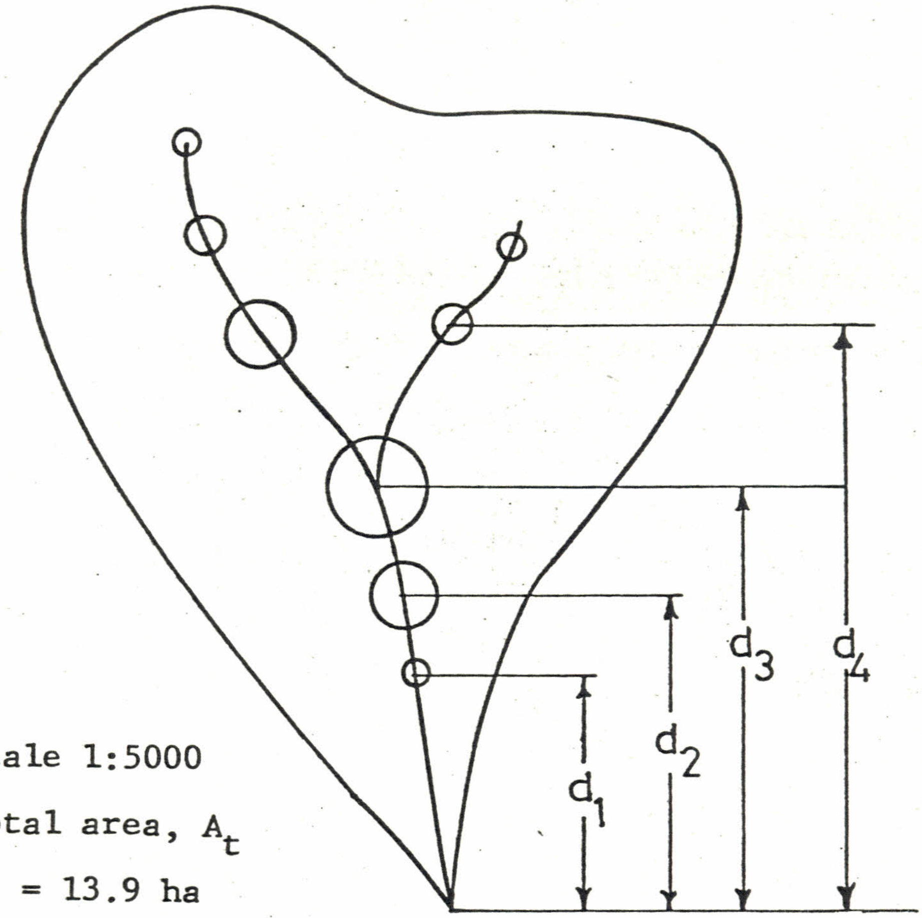
where R_u is the seasonal runoff (m) expected from the catchment area.

2.2.3. (B) CONSTRAINED SMALL RESERVOIR IRRIGATION SYSTEM OPTIMIZATION

A small reservoir can be placed at various topographically suitable locations on drainage ways in an agricultural watershed. The size (storage capacity) of a small reservoir has two constraints which vary with its location, (Fig. 3). It should not be greater than the available runoff i.e. supply of water (which will increase as the tank is moved toward the mouth of the watershed). Also the effective volume of the tank should not be greater than amount of water than can be used 'down stream' from the tank i.e. irrigation demand. (This amount will decrease as the tank is moved towards the mounth of the basin). This case is being called 'constrained' because of the above restrictions to be met by the optimum location, storage capacity and dimensions of a small reservoir.

The first step is to find the functional relationships of reservoir storage capacity, V_t with respect to the distance d, the tank is located from the mouth of the basin, i.e.:

$$\phi(d) = \min\{h(d), k(d)\} \tag{24}$$



Scale 1:5000

Total area, A_t

= 13.9 ha

FIG 3. : ALTERNATE LOCATIONS OF A TANK IN A HYPOTHETICAL SMALL WATERSHED.

where,

d = distance of a tank from the mouth of the watershed.

$\Phi(d)$ = the functional relationshipe of total volume V_t with respect to distance d.

$h(d)$ = functional relationship between runoff and distance d.

$k(d)$ = functional relationship between irrigable land 'downstream' and the distance d.

This formualtaion is for irrigation by gravity but if the value of water is such that it is economically feasible to pump water upstream for irrigation then pumping can also be included. The value of water can be found by an indirect method such as derived demand as suggested by Helweg and Sharma (1982). If pumping of water upstream is justified, then;

$$\Phi(d) = \min\{h(d), k(d) + P_e\} \tag{25}$$

where,

P_e = irrigation demand of the economic pumping area upstream, m^3

For gravity irrigation, at an optimum location d, equation 24 can be written in general as:

$$h(d) = k(d) \tag{26}$$

h(d) and k(d) can be determined as:

$$h(d) = R_u A''_c - S_p - E_v$$

where,

R_u = seasonal runoff expected from catchment area, m

A''_c = optimal catchment area upstream from the reservoir, m^2.

and

$$k(d) = \frac{I}{\eta} Ai$$

where,

I = irrigation requirements of the crops downstream, m

η = project irrigation efficiency

Ai = optimum irrigable area, m^2

By substituiting these values of h(d) and k(d) in equation (26)

$$R_u A''_c - S_p - E_v = \frac{I}{\eta} Ai$$

$$\text{or} \quad Ai = \frac{R_u A''_c - S_p - E_v}{I/\eta} \quad (27)$$

Also at a location where harvested water exactly meets the demand of all the area below it (Ai_e) in a basin:

$$A''_c = A_t - Ai_e$$

thus

$$R_u(A_t - Ai_e) - S_p - E_v = \frac{I}{\eta} Ai_e$$

where,

A_t = total area of the basin, m^2

Ai_e = all area of the basin below the small reservoir located such that it exactly meets the irrigation demand, or equilibrium irrigation area, m^2

or,

$$Ai_e = \frac{(R_u A_t - S_p - E_v)}{(R_u + I/\eta)} \tag{28}$$

The area Ai_e is not the optimum area available for irrigation because the net benefits need not necessarily be maximum at this location of reservoir due to nonlinear nature of various cost functions. In general the area available for irrigation below a small reservoir on a basin may be either; (i) more than what can be irrigated by harvested water, or (ii) exactly equal to the area, that can be irrigated by the tank Ai_e, as in equation (28) or (iii) can be less than the area, that can be irrigated by a tank. The relationship of the distance from mouth of a basin, d, and area of the basin, A_d up to distance d for a typical watershed shown in Fig. 3 (scale 1 : 5000) has been plotted in Fig. 4 as curve AEB. This curve has been determined by simply finiding the area up to various locations d_1, d_2, ... and will vary according to the shape of the basin. In this case a cubic regression equation was easily fitted by using A_d as a dependent variable and d as an independent variable. At a typical location L, on the curve AEB in Fig. 4, KL is the catchment area from which water can be harvested into the tank and LN is all the area below it that is available for irrigation but due to

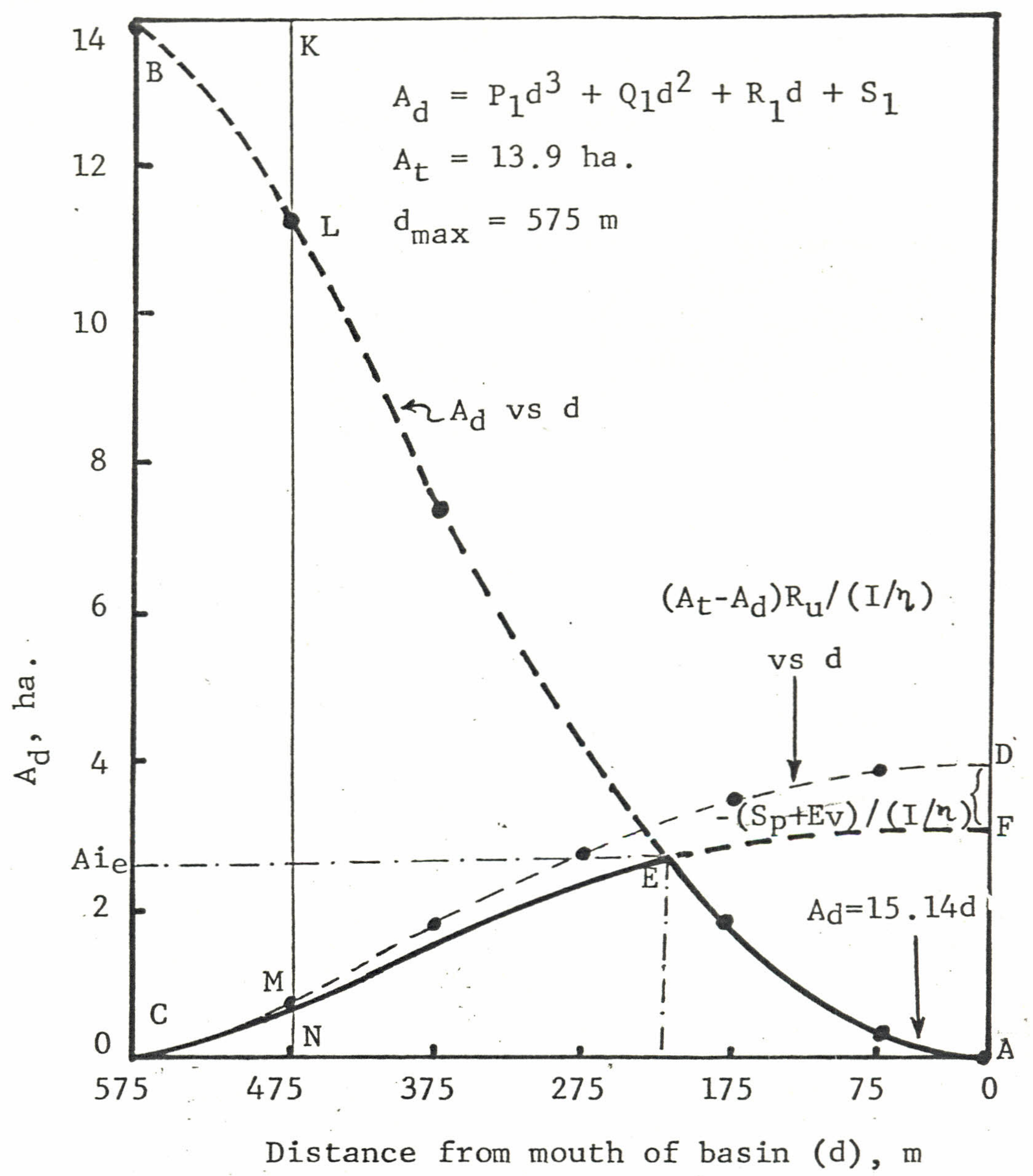


Fig. 4 : The relationship between distance from mouth of a basin (d) and area upto d (A_d) - for a typical small basin in Fig.3.5.

limited runoff $(A_t - A_d)R_u$ available only area MN = $(A_t - A_d)R_u/(I/\eta)$ can be irrigated by the capacity of the tank if seepage and evaporation losses are ignored. The curve CMD can be obtained in this manner by knowing the A_d versus d relationship. Part of this area depicted by curve CMD cannot be irrigated due to the volume of harvested water lost by seepage and evaporation $\{= -(S_p + E_v)/(I/\eta)\}$. Thus the area that can be irrigated can be shown as curve CEF which is having a shift from curve CMD by an amount equal to $-(S_p + E_v)/(I/\eta)$. Since seepage and evaporation losses are functions of the size of the tank, these losses cannot be calculated straighforwardly but are calculated in the model during the process of optimization. It is obvious that the location E on curve AB is the location where harvested water just meets the irrigation demands of all the area (Ai_e) below it. Thus the area that can either be irrigated, CE (which is constrained by the storage capacity of a reservoir) or that is available for irrigation, EA (which is constrained by the location of a small reservoir) is the area on the curve CEA. The A_d versus d curve for the typical example of a basin in Fig. 3., can be represented by the following polynomial regression equation:

$$A_d = -0.00092d^3 + 1.07995d^2 - 76.62125d + 1197.42878 \quad (29)$$

where A_d is in square meters and d in meters. This relationship in general accurately predicts the area A_d, as a function of distance d (for tha small basin in Fig. 3), except for very low values of d (at d less than 75 m). Hence the following straight line equation for $d \leq 75$ m was used:

$$A_d = 15.14\ d \text{ for } d < 75 \quad (30)$$

Now, in equation (27), we can determine the irrigable area limited by the capacity of a tank (curve CE in Fig. 4) as:

$$Ai_1 = \frac{(A_t - A_d)R_u - S_p - E_v}{(I/\eta)} \tag{31}$$

and area that is limited by its availability (curve EA in Fig. 4) upto Ai_e as:

$$Ai_2 = A_d \tag{32}$$

Thus now we can generalize; the optimum area to be irrigated by a tank:

$$Ai = Ai_1 \qquad \text{if } Ai \leq Ai_e \tag{33}$$

and $$Ai = Ai_2 \qquad \text{if } Ai > Ai_e \tag{34}$$

where,

$$Ai_2 = A_d \text{ in equation (29)}$$
$$\text{if } d_{Max} \geq d \geq 75 \tag{35}$$

here,

d_{Max} = total length of the basin (= 575 in Fig. 3)

$$Ai_2 = A_d \text{ in equation (30)}$$
$$\text{if } d < 75 \tag{36}$$

and $Ai_2 = 0 \text{ if } d = 0$ (37)

By now the optimum irrigation area Ai has been determined. The optimum total storage capacity or size of a reservoir V_t'', can now be determined by knowing the optimum catchment A_c'' from equation (27) as:

$$A_c'' = \frac{Ai(I/\eta) + S_p + E_v}{R_u} \quad (38)$$

and hence optimum tank size:

$$V_t'' = R_u . A_c'' \quad (39)$$

The objective of 'constrained' small reservoir irrigation system optimization can now be defined as maximization of net benefits (or minimization of negative value of net benefits) exactly as in the 'unconstrained' case (equation 16) except that now the total capacity of the reservoir in equation (18) is limited by the irrigation area in equations (33) and (34) which introduces a new decision variable 'd' into the model. Thus the 'constrained' optimization model can be formulated as:

$$\underset{\underline{r_1 h_1 h_2 d}}{\text{M I N}} \; - \{B_p \, (V_t'' - S_p - E_v) - (1.05C_u V_1 + C_L A_R)\} \quad (40)$$

Subject to:

$$V_1 = V_2 \quad (41)$$

$$V_1 + V_3 = V_t'' \quad (42)$$

where,

$$V''_t = R_u A''_c$$

$$= Ai(I/\eta) + S_p + E_v$$

$$r_1, h_1, h_2, d \geq 0 \qquad (43)$$

If h_1 or h_2 are restricted due to soil depth (SD) or availability of gravity drop (GD) for intercepting ruoff at a higher location above the reservoir, these limitations can be imposed here too as done in the 'unconstrained' case in equations (21) and (22).

This model will give the optimum location d, optimum catchment area A''_c, the optimum storage capacity V''_t, and optimum dimensions r_1, h_1 and h_2 of a small reservoir for the 'constrained' case.

The Optimum Storage Capacity, Location and Dimensions of the Small Reservoir Irrigation System

The optimum location and storage capacity of a small reservoir, irrespective of being 'unconstrained' or 'constrained', are the minimum of these two. (This is because if 'unconstrained' case gives maximum benefits for lesser capacity than the reservoir system design is not a 'constrained' case. This can be determined only after solving both cases.) The optimum location is decided by the optimum catchment, A^*_c from equations (23) and (38):

$$A_c^* = \min \begin{cases} A_c' \\ A_c' \end{cases} \qquad (44)$$

and the optimum storage capacity of a small reservoir, V_t^*, from equations (20) and (39) is;

$$V_t^* = \min \begin{cases} V_t' \\ V_t'' \end{cases} \qquad (45)$$

The optimum dimensions of a small reservoir r_1^*, h_1^* and h_2^* are those which give optimum storage capacity V_t^* in equation (45).

Depending on the optimum storage capacity and location, the optimum command area of the reservoir can be determined. On relatively large catchment or on regional basis, the 'unconstrained' criteria will govern the planning of a system of irrigation. This criteria can thus be used to ascertain how many small reservoirs are needed and where they are to be located, in order to derive maximum beneficial use of the system for uplifting the agriculture in rainfed regions. In section 4 it is shown that the unconstrained irrigation system design is usually optimal.

3. THE NONLINEAR PROGRAMMING ALGORITHM

The problem of optimization of small reservoir irrigation system evidently is a problem of optimization of a nonlinear function subject to nonlinear equality constraints. A complete review of various nonlinear programming algorithm including their suitability to solve the proposed model have been conducted by Sharma (1981). It was found that the constrained Flecher - Powell (CONMIN) algorithum developed by

Haarhoff, Buys and Molendorff (1969) is capable of solving the proposed model. The CONMIN algorithm was modified to suit the specific requirements of the model for optimization of small reservoir irrigation systems. For obtaining a copy of the computer listing (in Fortran IV) of the proposed model and its logic diagram along with the modified CONMIN algorithm, the reader is advised to refer to Sharma (1981). Hence forth this model is referred to as Small Reservoir Irrigation System Optimization (SRISO. FTN) Model.

4. RESULTS AND DISCUSSIONS:

For demonstration of the results of the model an example for optimizing the small reservoir irrigation system for Hyderabad region in South India is presented here. The input data used for this example is given in Table 1. Average seepage and evaporation rates of 5 mm/day from a reservoir are assumed. The future benefits of the irrigation system over the life of reservoir (assumed 25 years) are discounted at an annual discount rate of 6%. The effect of increase in seepage rates and discount rates are discussed in detail later. The irrigation system is planned to provide 28 cm of irrigation to post-monsoon season sorghum crop. The nature of yield response to water use for this crop is given by equation (6). All the water in the reservoir is expected to be used for irrigation within 120 days of filling (October to February). The cost of land to be occupied by the reservoir is assumed to cost Rs.2500/ha. For the 'constrained' case let us further assume that the size of basin (A_t) is only e.g. 13.9 ha as for the small basin in Fig. 3. Also assume that the runoff available from this catchment is 100 mm/ha (Table 1). The area-distance (d) relationship for this catchment is given by equations (33) and (34).

Tabel 1. Input data for the example solved by the small reservoir irrigation system optimization model SRISO.FTN (for Hyderabad in Semi-Arid India)

Input Data	Value
P	Rs.97/100 Kg, (Price of Sorghum)
Q	28 cm, (Quantity of irrigation delivered to field)
CI	Rs.576/ha, (Fixed cost of inputs for sorghum)
L	Rs.13.5/ha, (Cost of irrigations)
N_o	3, (No. of irrigations)
S	5 mm/day, (Seepage rate from reservoir)
E	5 mm/day, (Evaporation rate from reservoir)
N_p	120, (No. of days for emptying the reservoir)
N	25 years, (Life of a reservoir)
C_L	Rs.2500.00/ha, (Cost of land)
R_u	100 mm/ha, (Seasonal runoff expected)
A_t	13.9 ha, (Total area of a basin)
η	0.8, (Project irrigation efficiency)
d_{max}	575 m, (Maximum length of basin)

4.1. THE OPTIMUM SMALL RESERVOIR IRRIGATION SYSTEM:

Table 2 gives the solution of the above example by SRISO.FTN model for both 'unconstrained' and 'constrained' cases. From these two cases for the given example of 13.9 ha. basin in Fig. 3 it is evident from Table 2 that the optimum small reservoir irrigation system is the 'unconstrained' system since its storage capacity and catchment area is less than the 'constrained' case. Hence for the example, the optimum irrigation system has:

Optimum size $V_t^* = 1.05478$ ha m

Optimum catchment (location) $A^*_{catch} = 10.547$ ha

Optimum dimensions

$$r_1^* = 39.76 \text{ m}$$

$$h_1^* = 0.61 \text{ m}$$

$$h_2^* = 1.46 \text{ m}$$

From this example we can generalize that most often, except for very small basins (e.g. < 10 ha), the unconstrained case will give the optimum small reservoir irrigation system. Hence the 'unconstrained' SRISO.FTN model is to be used for planning of the small reservoir irrigation system if the system is desired on larger catchments. The optimum solution of the SRISO.FTN which gives optimum storage capacity, optimum location (catchment) and optimum dimensions can be used to predict how many reservoirs are required and where. Within the general area predicted by the model for locating a small reservoir, topographically favorable spots are to be preferred.

Table 2. Optimum small reservoir irrigation system for Hyderabad region in India for post-monsoon irrigation of sorghum crop (input date as per Table 1) on Alfisols.

Given: Top width of Dyke = 2 m, Free Board = 1 m, Dyke Slope = 1:1
Initial Guesses: r_1 = 40 m, h_1 = 2 m, h_2 = 1 m.

Optimum Design Variables	Unconstrained Case	Constrained Case
Size (V_t) ha m	1.05478	1.18270
Upstream Catchment Area (A_{catch})	10.547	11.8270
r_1, m	39.76	33.38
h_1, m	0.61	1.13
h_2, m	1.46	2.16
V_1, m^3	2991.77	3843.51
V_2, m^3	2991.78	3854.68
V_3, m^3	7556.05	8078.21
Location d^*, m		195.95
Storage/Excavation (V_t/V_1)	3.53	3.07

*Distance from mouth of the basin for Fig. 3. (example)

Another general question often posed is whether it is advisable to build many very small reservoirs (ponds) or one very large reservoir? The model answers this question. Any reservoir size below or above the optimum storage capacity (size) is not advised for a given set of topographic, hydrologic, climatologic and agronomic conditions. For the example above reservoir size other than 1.05478 ha. m is below optimum and not adivised to be built. However, large dams can become optimum on topographically favorable locations.

From the previous experience of the author at the International Crops Research Institute for Semi-Arid Tropics, Hyderabad, India (Sharma and Kampen, 1975; Sharma and Kampen, 1976; Sharma and Kampen, 1977) the final optimum dimensions obtained ($r_1* = 39.76$ m, $h_1* = 0.61$ m and $h_2* = 1.46$ m) are very practical from the reservoir construction and its use point of view because (i) with $h_1* = 0.61$ m only (depth of tank below ground level) it will be very easy to deliver water by gravity for crop irrigation and (ii) with $h_2* = 1.46$ m (height of tank above ground level) and given the rolling topography (2-5%) of Hyderabad (India) area along with the steep drops found below property boundaries in this area it will be very convenient to intercept runoff at an elevation of 1.46 m from the reservoir location within reasonably small distance from the small reservoir.

4.2. SENSITIVITY ANALYSIS:

Of the input to the design model, seepage and discount rate are most uncertain. Evaporation is fairly stable a given location as are costs, product prices, and other variables. Consequently a sensitivity analysis was conducted on those two inputs. Figures 5 and 6 indicate the results.

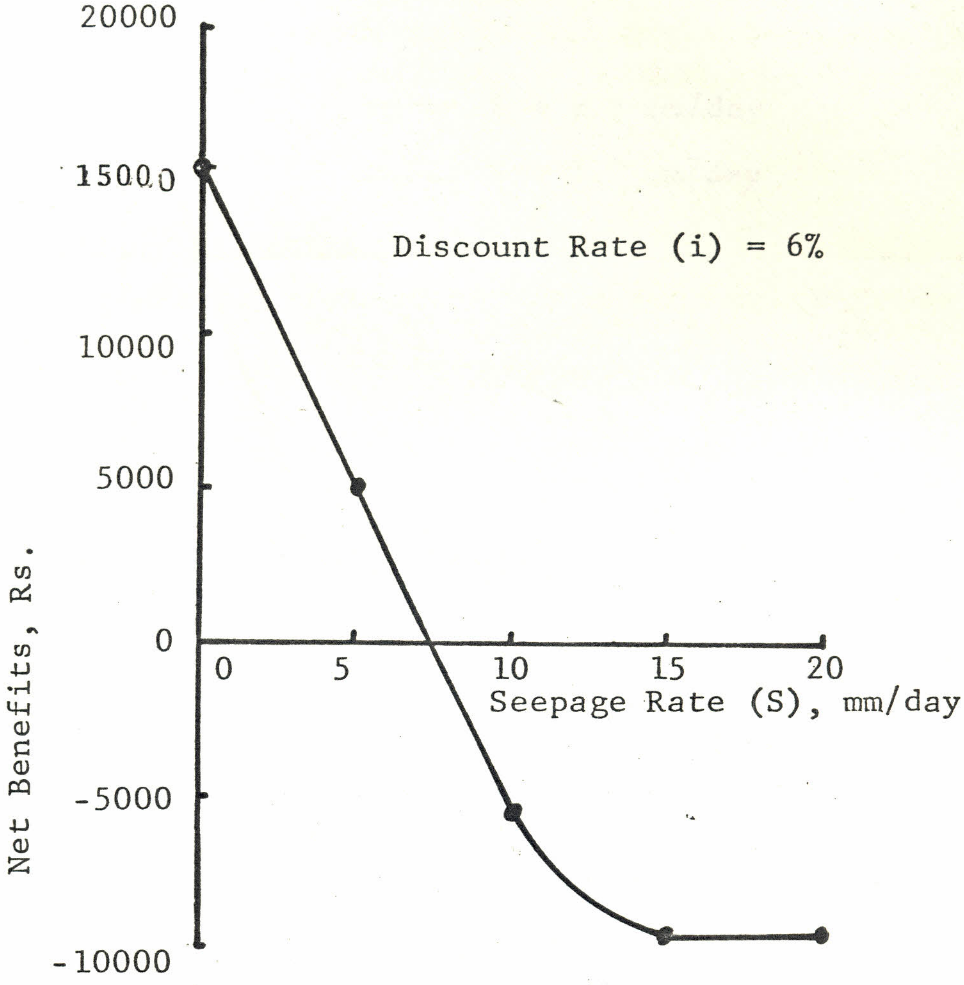


Fig. 5 : Effect of seepage rate (S) variation on net benefits of the small reservoir irrigation system.

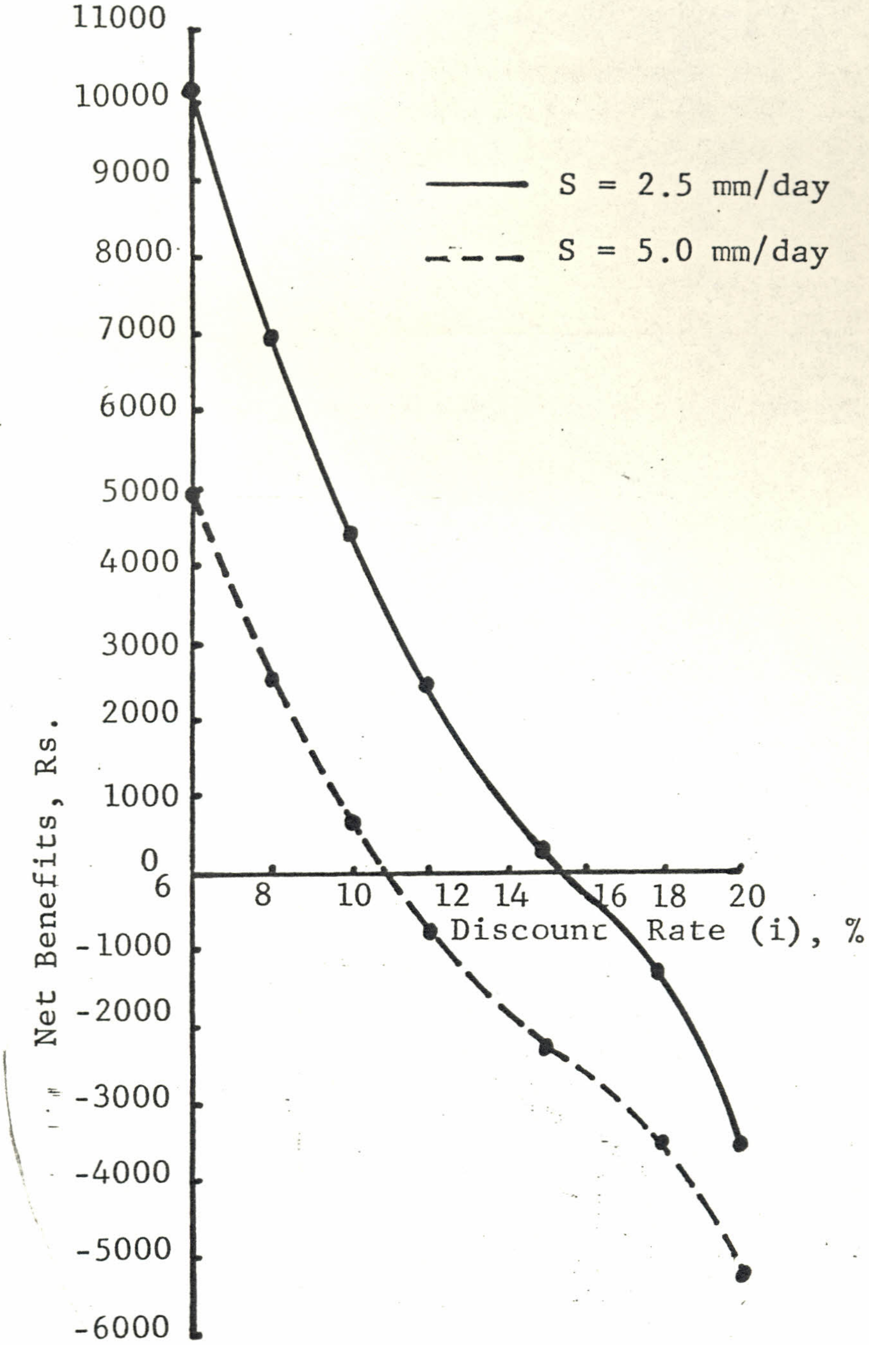


Fig. 6. : Effect of increase in discount rate (i) on net benefits of the irrigation system at two levels of seepage rates (S) in the reservoir.

Both these figures demonstrate that investiment in seepage control will increase net benefits substanially particularly if the seepage rate and discount rates are high.

5. CONCLUSIONS

This work shows that in most cases, the unconstrained design model is sufficient. The optimal design of these tanks show promise of significant increases in efficiency measured either by storage to excavation ratio or by net benefits.

The final optimization code was run on a LSI 11/23 mini computer indicating that it can be utilized in any area with minimal computational capability. Of course engineering judgement should modify the computer solution when appropriate.

6. APPLICABILITY FOR NORTH-EAST BRAZIL:

The model can directly be used for planning and designing small reservoir irrigation system in North-East Brazil. In case the small reservoir is planned for Cattle e.g. in very arid areas or for both Cattle and irrigation e.g. in arid areas the livestock component should be added to the objective function. These modifications are in progress at CPATSA.

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